Instructions: Complete each of the following exercises for practice.

1. Use the chain rule to compute $\dfrac{df}{dt}$.
   (a) $f(x,y) = xy^3 - x^2y; \quad x(t) = t^2+1, \quad y(t) = t^2 - 1$
   (b) $f(x,y) = \dfrac{x-y}{x+2y}; \quad x(t) = e^{\pi t}, \quad y(t) = e^{-\pi t}$
   (c) $f(x,y) = \sin(x)\cos(y); \quad x(t) = \sqrt{t}, \quad y(t) = \dfrac{1}{t}$
   (d) $f(x,y) = \sqrt{1+xy}; \quad x(t) = \tan(t), \quad y(t) = \arctan(t)$
   (e) $f(x,y,z) = x\exp(\frac{y}{z}); \quad x(t) = t^2, \quad y(t) = 1-t, \quad z(t) = 1+2t$
   (f) $f(x,y,z) = \ln(\sqrt{x^2+y^2+z^2}); \quad x(t) = \sin(t), \quad y(t) = \cos(t), \quad z(t) = \tan(t)$

2. Use the chain rule to compute $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.
   (a) $z(x,y) = (x-y)^5; \quad x(s,t) = s^2t, \quad y(s,t) = st^2$
   (b) $z(x,y) = \arctan(x^2+y^2); \quad x(s,t) = s\ln(t), \quad y(s,t) = te^s$
   (c) $z(x,y) = \ln(3x+2y); \quad x(s,t) = s\sin(t), \quad y(s,t) = t\cos(s)$
   (d) $z(x,y) = \sqrt{x}e^{xy}; \quad x(s,t) = 1+st, \quad y(s,t) = s^2 - t^2$
   (e) $z(r,\theta) = e^r\cos(\theta); \quad r(s,t) = st, \quad \theta(s,t) = \sqrt{s^2+t^2}$
   (f) $z(u,v) = \tan(\frac{u}{v}); \quad u(s,t) = 2s+3t, \quad v(s,t) = 3s-2t$

3. A function $f(x,y)$ with $\mathrm{dom}(f) = \mathbb{R}^2$ is *n-homogeneous* when $f(tx,ty) = t^n f(x,y)$ for all t (where $n > 0$).
   (a) Verify that $g(x,y) = x^2y + 2xy^2 + 5y^3$ is 3-homogeneous.
   (b) Assume f is n-homogeneous and has continuous second-order partial derivatives. Prove the following.
      i. $x\,\dfrac{\partial f}{\partial x} + y\,\dfrac{\partial f}{\partial y} = nf(x,y)$
      ii. $x^2\,\dfrac{\partial^2 f}{\partial x^2} + 2xy\,\dfrac{\partial^2 f}{\partial x\,\partial y} + y^2\,\dfrac{\partial^2 f}{\partial y^2} = n(n-1)f(x,y)$
      iii. $\dfrac{\partial}{\partial x}[f(tx,ty)] = t^{n-1}f_x(x,y)$

4. Compute the gradient of the function.
   (a) $f(x,y) = \dfrac{x}{y}$
   (b) $f(x,y) = x^2\ln(y)$
   (c) $f(x,y,z) = x^2yz - xyz^3$
   (d) $f(x,y,z) = y^2e^{xyz}$

5. Compute the directional derivative $D_{\mathbf{u}}f$ at point P in the direction of angle θ.
   (a) $f(x,y) = xy^3 - x^2; \quad P = (1,2), \quad \theta = \frac{\pi}{3}$
   (b) $f(x,y) = y\cos(xy); \quad P = (0,1), \quad \theta = \frac{\pi}{4}$
   (c) $f(x,y) = \sqrt{2x+3y}; \quad P = (3,1), \quad \theta = -\frac{\pi}{6}$

6. Find the directional derivative of f at the point P in the direction of $\mathbf{v}$.
   (a) $f(x,y) = \dfrac{x}{x^2+y^2}; \quad P = (1,2), \quad \mathbf{v} = \langle 3,5\rangle$
   (b) $f(u,v) = u^2e^{-v}; \quad P = (3,0), \quad \mathbf{v} = \langle 3,4\rangle$
   (c) $f(x,y,z) = x^2y + y^2z; \quad P = (1,2,3), \quad \mathbf{v} = \langle 2,-1,2\rangle$
   (d) $f(r,s,t) = \ln(3r+6s+9t); \quad P = (1,1,1), \quad \mathbf{v} = \langle 4,12,6\rangle$